

A review of the housing market-clearing process in integrated land-use and transport models

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Abstract: The land-use/transport interaction (LUTI) modeling framework has become the current state of best practice for analyzing the interdependency between the land-use and transportation systems. This paper presents a comprehensive review of the housing market-clearing mechanisms used in operational LUTI models. Market clearing is a critical component of modeling housing markets, but a systematic review and critique of the current state of the art have not previously been undertaken. In the review paper, the theoretical foundations for modeling household location choice are reviewed, including bid-rent and random utility theories. Five LUTI models are discussed in detail: two equilibrium models, MUSSA and RELU-TRAN, and three dynamic disequilibrium models, UrbanSim, ILUTE, and SimMobility. The discussion focuses on the following key points: the assumptions embedded in the models, the aggregation level of households and locations, computational cost and operationalization of the models. One of the challenges is that there are rarely any empirical studies that compare the performance of equilibrium and dynamic models in the same study context. Future research is recommended to empirically investigate the pros and cons of the two modeling approaches and compare the model performances for their representativeness of real-world behavior, computational efficiencies, and abilities for policy analysis. More sophisticated studies about the impacts of agents' behavior on the housing market-clearing process are also recommended.

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1 Introduction

It is commonly believed that there is a two-way interaction between land-use and transport. Land-use distribution defines locations for human activities, which generate trips in the transportation system. The transportation infrastructure creates opportunities to make such

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trips and, by providing accessibility to land, can influence location decisions (Wegener & Fuerst, 2004). The interdependency between land-use and transport inspires the development of land-use/transport interaction (LUTI) models, which incorporate feedback between the land-use and transport systems. Over the past decades, generations of LUTI models have been developed, from the earliest spatial-interaction models to the latest micro-simulation models (Cordera et al., 2017).

Most of the existing LUTI models are comprised of three major integrated components: demographics, land use, and travel demand (Acheampong & Silva, 2015). The demographic component includes households' socioeconomic variables that can impact their location/relocation choices and travel behaviors. The travel demand component usually contains the four-step approach to model trip generation, distribution, modal split, and network assignment (Acheampong & Silva, 2015). Some recent LUTI frameworks incorporate activity-based travel demand models. The land-use component generally refers to the detailed sub-models of the urban land market, such as modeling of residential, firm, and other actors' location choices (Lopes et al., 2019). In particular, the housing market sub-model is one of the most important sub-models in all operational LUTI models. It contains various aspects of households' location/relocation process, from modeling the search process and residential location choices to clearing the housing market (Habib, 2009). Among them, a very essential but often overlooked part is the housing market-clearing process. It defines how and at what prices the households active in the market are matched with available locations (Farooq & Miller, 2012; Hurtubia et al., 2019). Although it is a standard component of the existing LUTI models, very few spend much effort examining how representative the market-clearing mechanisms are of real-world behavior.

This paper aims to provide a comprehensive review of the housing market-clearing mechanisms used in existing LUTI models. Although various mechanisms are utilized in different models, they generally belong to one of the two major approaches: equilibrium and dynamic disequilibrium. This paper reviews the theoretical foundations behind the housing market-clearing mechanisms and their applications in equilibrium and dynamic disequilibrium LUTI models. There are several review papers on LUTI models in the literature (e.g., Acheampong & Silva, 2015; Hunt et al., 2005; Moeckel et al., 2018; Wang & Wu, 2010) and studies that include brief discussions of housing market-clearing mechanisms (e.g., Farooq & Miller, 2012; Hurtubia et al., 2019). However, to the best of our knowledge, there has not been a review paper that specifically focuses on the different housing market-clearing mechanisms utilized in LUTI models and includes detailed discussions on their strengths, limitations, and future research challenges and directions. This review paper fills this gap and provides a reference for researchers interested in this topic.

The remainder of this paper is organized as follows: Section 2 lays the foundation for the discussion by summarizing the key theoretical foundations for modeling residential location choices. This is followed in Section 3 by a literature review of the housing market-clearing mechanisms applied in state-of-art LUTI models that build upon the theoretical foundations discussed in Section 2, highlighting their advantages and drawbacks. Then, Section 4 offers a discussion of the major criticisms surrounding the equilibrium and dynamic disequilibrium approaches used for clearing housing markets. The discussion focuses on the assumptions, degrees of aggregations, computational costs, and operationalizations of the two approaches, as well as future research directions. The effects of transportation accessibility on housing valuation and the market-clearing process are also discussed in Section 4. Finally, Section 5 concludes the paper with a summary of the current status, challenges, and potential improvements of market-clearing mechanisms.

2 Theoretical foundations of modeling residential location choices

Over the past decades, several theories have been developed to investigate and predict residential location choices. The foundation for modeling urban land markets can be credited to Alonso's bid-rent theory (Alonso, 1960). He assumed a monocentric city where the distance to the city centre is the only determining factor for land prices and then modelled the bidding process between buyers and sellers. The land market is assumed to be an auction market, where the potential buyers try to outbid each other for a given location, and the location is assigned to the highest bidder. In his model, an equilibrium bid-rent surface exists for each land-use type, such as residential, business, etc. (Alonso, 1960). A balance between land supply and demand is achieved at the equilibrium prices. Later, the modeling of residential location choices devolved into two main approaches: the household location choice approach and the bid-auction approach (Hurtubia, 2012; Martínez, 2018). The following subsections present the theoretical formations of the two approaches and a bid-choice location framework that attempts to unify them.

2.1 Household location choice approach

Lerman (1976) and McFadden (1977) applied random utility theory and discrete choice analysis to understand residential location choices. This seminal work soon became one of the two dominant approaches for modeling residential location choices and is generally known as the choice approach. This approach assumes that a household chooses the residential location that can maximize its utility, given that a price has been set for that location. A discrete choice model is utilized to model residential location choices, in which household characteristics and locational attributes can all be used as explanatory variables. Since households are assumed to be price takers, one of the explanatory variables has to be the land (or housing) price. The general form of the random utility model of location choice is shown below:

$$U_{ih} = V_{ih} + \varepsilon_{ih} \quad (1)$$

where,

U_{ih} = the utility of household h obtained from location i ,

$V_{ih} = \beta X_{ih}$ is the systematic utility computed using explanatory variables X_{ih} and parameters β ,

ε_{ih} = the random or unobserved utility.

If ε_{ih} is assumed to be identically and independently distributed (IID) following a Type I Extreme Value distribution (i.e., Gumbel distribution), then the probability of household h choosing location i is computed by the multinomial logit (MNL) model shown below:

$$P_{i|h} = \frac{e^{\mu V_{ih}}}{\sum_{i'=1}^L e^{\mu V_{i'h}}} \quad (2)$$

where L is the set of locations available for household h , and μ is a constant scale that is normalized to one in value when only one dataset is used to estimate the MNL model.

A common concern about the choice approach is determining land prices and the price endogeneity problem. Theoretically, the land prices should be determined endogenously due to the interaction between supply and demand, usually through a market-clearing process. However, a common practice to estimate land prices is to utilize hedonic price models, first

proposed by Rosen (1974). Hedonic price models usually take the form of a regression model and describe land prices using locational attributes. Since the market-clearing process is not considered in hedonic price models, they may be insensitive to the changes in market conditions and the heterogeneity in households' preferences (Hurtubia et al., 2010). Moreover, the locational attributes used to compute hedonic pricing may be correlated to the explanatory variables in the utility function, which can cause the endogeneity problem in residential location choice models. Such a problem can result in unintuitive price coefficients in residential location choice models, such as being small, positive, or statistically insignificant (Guevara & Ben-Akiva, 2006).

2.2 Bid-auction approach

Shortly after Lerman (1976) and McFadden (1977), Ellickson (1981) applied random utility theory to Alonso's bid-rent theory and developed a stochastic bid-rent function. Ellickson's work lays the foundation for the bid-auction approach, the other dominant approach for modeling residential location choices. This approach assumes an auction market in which a household bids its willingness to pay for different locations, conditional on the fixed levels of utility that the household expects to achieve. For a given location, the auction simultaneously determines its buyer and sales price by assigning it to the highest bidder and taking the maximum bid as its price. Ellickson's (1981) stochastic bid-rent function defined the bid of a household h on a location i , represented by \overline{B}_{hi} , as a function of locational attributes z_i and a random error term ε_h :

$$\overline{B}_{hi} = B_{hi} + \varepsilon_h \quad (3)$$

where, $B_{hi} = B_h(z_i)$ is the systemic bid of household h for the location i with attributes z_i . The random error term ε_h captures the unobserved heterogeneity in households' preferences. It is different from the random error term in the choice approach that represents unobserved characteristics of the location (Hurtubia, 2012). If ε_h is assumed to be IID following a Type I Extreme Value distribution, the probability of household h being the highest bidder for location i can be expressed as the following MNL model:

$$P_{h|i} = \frac{e^{\mu' B_{hi}}}{\sum_{h'=1}^H e^{\mu' B_{h'i}}} \quad (4)$$

where H is the set of households bidding for location i and μ' is the scale parameter. The stochastic bid-rent function can be understood as the probability of location i choosing household h , which is the dual of the choice approach model. The expected price of the location can be directly estimated from the stochastic bid-rent function as the expected maximum bid, which is the log-sum of all bids (Rose & Martínez, 2007):

$$p_i = \frac{1}{\mu} \ln \left(\sum_{h'=1}^H e^{\mu' B_{h'i}} \right) + \frac{\gamma}{\mu} \quad (5)$$

where γ is the Euler's constant. The term $\frac{\gamma}{\mu}$ is to account for the fact that the bids can only be identified up to an unknown constant, and the absolute value of the prices cannot be estimated from the relative bids (Hurtubia, 2012). The bid-auction approach considers the

interaction between supply and demand in its land price formation process, which makes it more sensitive to market conditions than the choice approach.

2.3 Bid-choice location framework

Many studies have attempted to build upon Ellickson's stochastic bid-rent function, among which, arguably the most notable, is Martínez's bid-choice location framework (Martínez, 1992; Rose & Martínez, 2007). It is a unified microeconomic framework that combines the choice and bid-auction approaches. The fundamental argument of the framework is that the urban land market is a case with common values. The goods (i.e., locations) are quasi-unique with known values that can be estimated from similar alternatives sold. This argument ensures an auction process with the final price determined by the highest bid, allowing buyers to "behave as close to price takers as desired" (Rose & Martínez, 2007). The analytical derivation of this framework is explained as follows. The maximum utility that household h can obtain from location i is formulated as $V_h(I_h - r_i, P, z_i)$, where I_h is the income of the household, r_i is the price of the location, P is the vector of prices on composite goods, and z_i is a vector of locational attributes. Under the bid-auction assumption, the price of the location is the maximum value that the household is willing to pay for this location to achieve a given level of maximum utility U_h^* . Therefore, the household's willingness to pay WP_{hi} for location i can be determined by inverting V_h on land price r_i (Rose & Martínez, 2007):

$$WP_{hi} = I_h - V_h^{-1}(P, z_i, U_h^*) \quad (6)$$

In the auction market, the location is assigned to the household with the highest willingness to pay. This can be formulated as $\text{Max}_{h \in H} WP_{hi}$, where H is the set of households bidding for this location.

To apply WP_{hi} in the context of the choice approach, it can be understood as the price that the household would pay to obtain the utility level U_h^* . The household would be indifferent in choosing any alternative location because it would pay the same WP_{hi} to obtain the same utility level U_h^* anywhere (Rose & Martínez, 2007). Following the choice approach, the price of the location is exogenously determined as p_i . The consumer surplus CS_{hi} that the household can gain from the location is the difference between the indifference value (i.e., WP_{hi}) and the actual price (i.e., p_i). Assuming maximizing consumer surplus is the same as maximizing utility, the household's optimal choice is to achieve the following:

$$\text{Max}_{i \in L} CS_{hi} = V_{hi} = WP_{hi} - p_i \quad (7)$$

where L is the set of alternative locations from which the household can choose. Now to combine the choice and the bid-auction approaches, the bid-choice framework proposes to replace the exogenous land price p_i with the maximum bid to obtain the following expression:

$$\text{Max}_{i \in L} CS_{hi} = WP_{hi} - (\text{Max}_{h \in H} WP_{hi}) \quad (8)$$

The above expression shows that when others outbid the household, the actual price becomes higher than its willingness to pay, and its consumer surplus becomes less than zero. So the maximum consumer surplus that the household can achieve is zero when it is the highest bidder at the location.

Martínez (1992) also performed a theoretical comparison between the choice and the bid-auction approaches and concluded that the two approaches are equivalent under equilibrium

conditions. This can be shown analytically through Bayes' theorem. Under the bid-auction assumption, the stochastic bid-rent function shown in Equation 4 can also be computed in terms of the household's willingness to pay WP_{hi} and the expected price of the location p_i :

$$P_{h|i} = \frac{e^{\mu' WP_{hi}}}{\sum_{h'=1}^H e^{\mu' WP_{h'i}}} = e^{\mu'(WP_{hi}-p_i+\gamma)} \quad (9)$$

As shown in Equation 7, the WP_{hi} and p_i can also be used to express consumer surplus, equivalent to utility in the choice approach. Therefore, the following expression can be obtained if the utility variable in the choice probability shown in Equation 2 is replaced with WP_{hi} and p_i :

$$P_{i|h} = \frac{e^{\mu V_{ih}}}{\sum_{i'=1}^L e^{\mu V_{i'h}}} = \frac{e^{\mu(WP_{hi}-p_i)}}{\sum_{i'=1}^L e^{\mu(WP_{hi'}-p_{i'})}} \quad (10)$$

If the scale parameters in the choice approach (i.e., μ) and the bid-auction approach (i.e., μ') are the same, the following expression can be generated by substituting Equation 9 into 10 (Hurtubia, 2012):

$$P_{i|h} = \frac{P_{h|i}}{\sum_{i'=1}^L P_{h|i}} \quad (11)$$

If the housing market is in equilibrium conditions, meaning supply equals demand, then $\sum_{i'=1}^L P_{h|i} = 1$ for every household in the market and $P_{i|h} = P_{h|i}$. Therefore, under equilibrium conditions, the two approaches are equivalent, and the residential location distributions estimated from the two approaches should be the same.

Martinez's work on the bid-choice location framework and the equivalence between the two approaches has significant impacts. The proof of bid-choice equivalence ends the dilemma of which approach to use. It assures that proper results on residential location distribution and price can be obtained from either approach. Moreover, the bid-choice location framework provides a way to unify the choice and bid-auction approaches and inspires the housing market-clearing mechanisms of several operational LUTI models, including MUSSA (Martínez, 1996; Martínez & Donoso, 2010), UrbanSim (Waddell, 2000), and ILUTE (Farooq & Miller, 2012). Nevertheless, the framework has been criticized for not considering the adjustment of housing supply when interacting with demand and its impact on housing prices (Waddell, 2000). Although housing supply is defined as exogenous in the equilibrium assumptions, Martínez (2018) explains that in the cases of excess supply or demand, buyers would increase or reduce equilibrium utilities. As a result, some sellers or buyers may exit the market, leading to the restoration of equilibrium and change in prices.

3 Housing market-clearing mechanisms in LUTI models

The interaction between residential location buyers and sellers occurs in the housing market. The goods traded in the housing market (i.e., housing units of various types) are believed to have a quasi-unique nature, meaning that they are similar but not equal (Farooq & Miller, 2012; Martínez, 2018). This is because they have differentiable locations that provide unique access to different amenities and built environments. In addition, different buyers have

different willingness to pay for the unique characteristics offered by each location, which also contributes to the quasi-unique nature of the housing units (Hurtubia et al., 2019). The location choice models discussed in the previous section are only the fundamental theories used to match locations and households. They do not account for the interaction between buyers and sellers, and the potential competition between buyers when supply is limited. Therefore, housing market-clearing mechanisms are developed to model the interaction between buyers and sellers, assign available housing units to buyers, and determine their sell prices. It is essential to solving the problem of “who gets which housing unit at what price” (Farooq & Miller, 2012). The market-clearing mechanisms generally differ between equilibrium and dynamic disequilibrium LUTI models. Descriptions of the two types of LUTI models are provided in the following subsections. Particularly, the market-clearing mechanisms of five LUTI models are reviewed in detail because they are representative mechanisms often utilized in the two types of LUTI models. The five models include two equilibrium models (i.e., MUSSA and RELU-TRAN) and three dynamic disequilibrium models (i.e., UrbanSim, ILUTE, and SimMobility). In addition, to facilitate a more comprehensive review, the housing market components of several other well-developed LUTI models are also introduced and summarized in Table 1.

Table 1. Summary of housing market components of existing LUTI models

LUTI Framework	Model Introduction	Housing Market Component
PECAS	Production Exchange Consumption Allocation System (PECAS) is a multiregional input-output model that simulates the integration between land-use and transport (Hunt & Abraham, 2005).	Residential location choices are modeled in its Activity Allocation (AA) module by matching households with available residential spaces. In its Space Development (SD) module, the prices for space are determined when an equilibrium is achieved between the flows of goods, services, and labor. Both residential and non-residential developments can be considered for the same space, and it would be assigned to the highest bidder in a bid-rent allocation.
TRANUS	TRANUS is a generic framework that models the integration between land-use and transportation (de la Barra et al., 1984).	For a given modeling period (e.g., a year), the productions and consumptions of a study area are estimated iteratively through its land-use and activity module to achieve an economic equilibrium (Dutta et al., 2012). This process includes the demand and supply of housing units. The housing prices are adjusted within the modeling period to clear the market when all available residential space is consumed by the households in demand.
ILUMASS	Integrated Land-Use Modeling and Transportation System Simulation (ILUMASS) is designed to be a full microsimulation model connecting land-use, transport and environment (Wagner & Wegener, 2007)	Households' relocation decisions are modelled in its residential mobility sub-module. Monto Carlo simulation is utilized to model them as transactions between buyers and sellers in the regional housing market (Moeckel et al., 2007). A household's relocation can be simulated either from the housing demand (i.e., buyers or renters) or the supply (i.e., sellers or landlords) side. However, no information is found on the explicit modeling of housing prices nor a market clearing process.

SILO	Simple Integrated Land-Use Orchestrator (SILO) is a microscopic land-use simulation system. It has three main modules: household relocation module, demography module, and real estate development module (Ziemke et al., 2016).	The housing market is represented by the real estate development and household relocation modules. The real estate development module simulates developers' decisions to demolish, renovate, and build new housing stocks. A hedonic price model is included to adjust the housing prices based on the supply and demand ratio. In the household relocation module, logit models are used to estimate a household's decision to move and relocation choices, while considering the household's travel and housing budgets and commuting time to workplaces (Moeckel, 2016; Ziemke et al., 2016). No information is found on the explicit modeling of a housing market clearing process.
MUSSA	MUSSA is a static equilibrium land-use model (Martínez, 1996).	The housing market component is developed based on Martínez's bid-choice location framework. A detailed review is presented in Section 3.1.1.
RELU-TRAN	RELU-TRAN is a spatial computable general equilibrium model (Anas, 2013).	The housing market component is a dynamic aggregated equilibrium model. A detailed review is presented in Section 3.1.2.
UrbanSim	UrbanSim is a land-use microsimulation with the ability to integrate with existing transportation models (Waddell, 2010).	The housing market component contains several sub-models that simulate household and employment mobility and location choices, real estate developments, and land prices. A detailed review is presented in Section 3.2.1.
ILUTE	ILUTE is an integrated full-feedback microsimulation model (Salvini & Miller, 2005).	Its housing market component employs a unique disequilibrium market-clearing approach that incorporates both game and random utility theories. A detailed review is presented in Section 3.2.2.
SimMobility	SimMobility is a multi-scale microsimulation platform (Adnan et al., 2016).	Its housing market module can simulate a daily bidding process between active buyers and sellers. A detailed review is presented in Section 3.2.3.

As shown in Table 1, it is worth noting that in some models, the housing prices are determined first either through a hedonic price model or demand/supply ratio, and then the demand is heuristically matched to the supply with a random sequencing of excess demand. In this case, the level of spatial disaggregation of the housing component can have a significant effect on the model output.

3.1 Housing market-clearing mechanisms of equilibrium models

There are generally two types of equilibrium models: static equilibrium and dynamic equilibrium models. Most of the equilibrium LUTI models adopt the static equilibrium condition, which assumes that supply, demand, and prices adjust instantaneously to achieve an equilibrium condition without external influences (Simmonds et al., 2013). This implies that a set of location choices and equilibrium prices maximize every household's utility, and sellers and buyers must have perfect information about the market to achieve this (Fujita, 1989; Hurtubia, 2012). The market is cleared when there are no unlocated households or unoccupied locations, and the equilibrium prices and location choices are determined. If the

choice approach is utilized, the market-clearing process would be to find the prices that can achieve an equilibrium between supply and demand. This process usually aggregates households and locations by their respective characteristics. Examples of LUTI models using such an approach include TRANUS (de la Barra et al., 1984) and PECAS (Hunt & Abraham, 2005). If the bid-auction approach is utilized, the market-clearing process would be to find the bids that allow all households to be located with maximum utility (Hurtubia et al., 2019). An example of this approach is the MUSSA model (Martínez, 1996; Martínez & Donoso, 2010).

The difference between dynamic and static equilibrium models is the role of external influences. Dynamic equilibrium models consider external influences, although their net effects cannot impact the ratios between system variables (Simmonds et al., 2013). RELU-TRAN (Anas, 2013; Anas & Liu, 2007) is believed to be the only operational dynamic equilibrium model.

The following subsections provide an overview of the housing market-clearing mechanisms from two equilibrium models: MUSSA and RELU-TRAN. These two models are selected because they each represent one type of equilibrium model and location choice approach. MUSSA is a static equilibrium model using the bid-auction approach, whereas RELU-TRAN is a dynamic equilibrium model using the choice approach.

3.1.1 Modelo de Uso de Suelo de Santiago (MUSSA)

The MUSSA model is a static equilibrium land-use model developed based on Martínez's bid-choice location framework. The theories of the framework have been described in Section 2.3. In the MUSSA model, households are aggregated based on socioeconomic characteristics, and housing units are categorized in terms of zones and dwelling types. The housing market is cleared at equilibrium, which can be achieved when two conditions are satisfied simultaneously: 1) each household in the market is assigned a housing unit that can maximize its consumer's surplus, and 2) each housing unit in the market is matched with its highest bidder (Martínez, 1996). The mathematical expression is presented in Equation 8. In addition, three assumptions are required for the market equilibrium: 1) all households are located somewhere, 2) supply is constrained to the available land in each zone, and 3) the supply of each dwelling type needs to comply with the historical tendencies (Martínez, 1996). The bid function of a household includes not only its willingness to pay for the housing attributes, as shown in Equation 3 but also an adjustment in the bid that would yield equilibrium. The price of a housing unit is defined by its expected maximum bid, which can be calculated using Equation 5. The probability of a household being the highest bidder of a housing unit (i.e., the location model) can be estimated from Equation 4.

In the newer generation of MUSSA (i.e., MUSSA II), constrained logit models are utilized to account for the constraints that housing buyers and sellers are subject to, such as income for buyers and regulations for sellers (Martínez & Donoso, 2010). Such constraints are represented by cut-off factors that are defined as binomial logit functions (Martínez et al., 2009). In summary, the equilibrium problem in the MUSSA model is solved through a system of nonlinear equations, whose solution includes the supply of housing units, the match between households and housing units, and their respective bidding prices.

Martínez & Donoso (2010) believe that with the addition of behavioral constraints, the MUSSA II model can be an effective tool for assessing various land-use and housing policies. They also argue against the interpretation that MUSSA is a path-independent model, and state that the interdependency between forecasting periods can be modelled using incremental multinomial logit functions. A dynamic extension to the MUSSA model was proposed by

Martínez & Hurtubia (2006) to use equilibrium to adjust housing prices under supply and demand surplus. The portion of the unlocated households and vacant housing units are estimated along with the housing prices for each modeling unit. However, gathering the detailed data required to calibrate this dynamic extension is often difficult, which prevents it from becoming an operational model (Hurtubia, 2012).

3.1.2 Regional Economy, Land Use, and Transportation Model (RELU-TRAN)

The RELU-TRAN model is a spatial computable general equilibrium model that simulates the interaction between the housing market, labor market, retail market, and travel behaviors (Anas, 2013; Anas & Liu, 2007). The housing market sub-model in RELU-TRAN is a dynamic aggregated equilibrium model that includes two types of agents: investors and consumers (i.e., households). Households are grouped based on their demographics, such as income, age, and ethnicity. Housing supplies are generally categorized based on their location and physical characteristics, such as the number of bedrooms. The basic assumptions of the model are listed below (Anas & Arnott, 1991, 1993):

- The modeling period is assumed to be a year.
- All agents are price takers, and there is no transaction cost.
- All housing units are available in the market every year.
- To clear the market, the expected demand and supply of the housing units must be equal.
- Investors can buy, rent, convert, and sell properties. They have perfect foresight about the costs and revenues of the properties, including future rents, asset prices, and conversion costs, such as demolition and reconstruction, etc.
- Households have two tenure choices: rent or own. Households who choose to buy the properties can convert them like investors. Each household has to choose one property to occupy every year.

In the RELU-TRAN housing market sub-model, investors become housing unit suppliers after buying and converting them. During this process, they are required to make three nested decisions: 1) bidding price for the initial purchase, 2) whether to rent or keep the housing units vacant before converting them, and 3) how to convert the housing units. The latter two are assumed to be independent and parallel decisions nested within the first bidding decision. After the initial purchase, the investors expect to maximize the sum of utility from the second and their decisions. The probabilities of renting and choosing different conversion activities can be calculated through binary logit and multinomial logit functions, respectively. The bidding price of the initial purchase is determined by the maximum bid under a competitive market assumption, which is achieved when the present worth of the expected net income of this investment equals zero (Anas & Arnott, 1991). The bidding prices are the equilibrium asset prices in the competitive market.

A household's choice of consuming a housing unit also has multiple stages: 1) whether to enter the housing market, 2) which group of housing units to select, and 3) which housing unit to choose from the selected group. In this case, the probability of a household choosing a particular housing unit is conditional upon the probabilities of choosing housing groups and the probability of entering the market, which can all be specified through a nested logit formulation (Anas & Arnott, 1991).

To ensure a dynamic equilibrium of the housing market, Anas and Arnott (1991) list three required conditions: 1) the same number of housing supplies and demands every year to clear the market, 2) the bidding prices equal the after-tax expected rate of return generated by the market-clearing rents, 3) the bidding prices need to give rise to the conversion activities that can lead to the market-clearing rents. The model is designed only to simulate the effects of exogenous change within a simulation period of T , during which a dynamic equilibrium is assumed. At the end of period T , a stationary equilibrium condition is applied as a terminal condition, meaning that housing supplies, asset prices, and rents would become stationary after period T (Anas & Arnott, 1993). The terminal asset prices can be used to compute the asset price vectors over the simulation years through backward recursion because the asset prices of the current year are a function of the current rents and asset prices in the next year. In summary, the modeling system is comprised of three unknown vectors for the simulation period: rents, asset prices, and housing supplies of each housing group. The variables related to demand and supply are given as inputs, and the associated choice probabilities can be determined through logit models. To simultaneously solve for the three unknown vectors, three equations are constructed: 1) asset prices are a function of rents and asset prices of other housing groups, 2) housing supplies are a function of demands, asset prices, rents, and probabilities of converting to different housing types, and 3) a market-clearing equation requiring zero excess demand. A nonlinear simultaneous solver is utilized to calculate the rents, asset prices, and housing supplies (Anas & Arnott, 1993).

As a dynamic equilibrium model, RELU-TRAN is advantageous in incorporating the effects of exogenous changes in the modeling period. However, it is still limited by the equilibrium assumptions, which can be unrealistic in representing the housing market.

3.2 Housing market-clearing mechanisms of dynamic disequilibrium models

Some researchers believe that imposing the assumptions of supply-demand equilibrium and agents with perfect information is an oversimplification of the housing market (Farooq & Miller, 2012). Dynamic disequilibrium models are developed to relax the equilibrium condition and introduce market dynamics through time. A common method to represent the temporal dynamics is to use recursive simulation models, in which the output of one period is utilized to adjust market behavior in the next period (Simmonds et al., 2013). The dynamic disequilibrium approach is widely utilized in microsimulation LUTI models. Unlike the aforementioned models that aggregate households and housing units into groups, microsimulation models simulate the behavior of agents at the level of individual households. The market-clearing process and determination of housing prices are generally simulated as the interaction between buyers and sellers based on certain assumptions about the market. In the following subsections, the market-clearing mechanisms from three dynamic disequilibrium microsimulation models are reviewed: UrbanSim (Waddell, 2000, 2002), ILUTE (Farooq & Miller, 2012; Salvini & Miller, 2005), and SimMobility (Adnan et al., 2016; Zhu et al., 2018). These are three popular microsimulation LUTI models that originate in North America. UrbanSim and ILUTE have been developed for over two decades, whereas SimMobility is a relatively recent agent-based microsimulation platform. Although both UrbanSim and ILUTE claim to be somewhat inspired by Martinez's bid-choice location framework, different assumptions about the agents' behaviors are made in their housing market-clearing mechanisms.

3.2.1 UrbanSim

UrbanSim is a land-use microsimulation model that originated in the United States to integrate with existing transportation models and assess the effects of land-use and transportation policies (Waddell, 2010). With the real estate market being the centre of the UrbanSim model system, it contains several sub-models that simulate household and employment mobility and location choices, real estate developments, and land prices (Waddell, 2002, 2010).

The residential location choice model in UrbanSim incorporates the bid component of Martinez's bid-choice location framework; however, it removes the equilibrium assumption and treats housing prices as exogenous to households' location choices. In other words, the households are assumed to be price takers, and the housing prices are estimated separately using a base-year hedonic price model and then updated for each simulation period. The model uses one year as a simulation period. It assumes that agents have imperfect information about the market and costs associated with searching and relocating to a new housing unit. Thus, the location choice decision can be decomposed into two stages: 1) the decision to relocate and 2) the choice of a housing unit.

Since the model does not solve an equilibrium problem, its market-clearing reconciles the housing demand and supply of each simulation period and adjusts the housing prices (Waddell, 2000). The steps and assumptions of its housing market-clearing process are summarized below (Waddell, 2000, 2010):

- Households' location choice probabilities are estimated using a multinomial logit model, as shown in Equation 10.
- The housing supply within a simulation period is assumed to be fixed. The market is cleared using a capacity-constrained algorithm that applies a "first-come, first serve" approach.
- The housing supply consists of existing vacant units, new development, and redevelopment projects from the most recent period. In the real estate development sub-model, the developers' choices of (re)development projects are based on the profitability expectations given historical prices, revealed preferences and demands. The locations and types of (re)development are estimated through MNL models (Waddell, 2002).
- The algorithm attempts to match each household with the housing unit that can provide the highest utility. However, if that housing unit has already been occupied, the household is forced to relocate to the housing unit that can achieve the second highest utility.
- The model assumes that the market is cleared once all households are located, and the housing prices are updated at the end of each simulation period based on the demand and supply ratio. The developers also respond to the updated housing prices to maximize the profitability expectations of future (re)development choices.

Such a market-clearing approach is justified as being more realistic than the equilibrium approach. In reality, agents do not have perfect information about the market, so they are likely to make decisions that minimize the search and transaction costs and the risks of missing out. For buyers, such a decision would be to choose the best available housing unit at the time; for sellers, it would be to sell the unit to the first buyer at the given price. However, this market-clearing mechanism has been criticized as ignoring market effects (Hurtubia, 2012). The housing prices are based solely on the base-year locational attributes when exogenously computed through a hedonic price model. Although the prices are adjusted later using a

supply-demand ratio, it cannot truly reflect the effects of supply or demand surplus on the housing prices.

Moreover, some researchers believe that the assumption of buyers as price takers is not necessarily true in the housing market (Farooq & Miller, 2012). This assumption oversimplifies the interactions between agents in the housing market, especially when a buyer-driven or seller-driven market exists. For example, the transaction prices may be higher than expected in a seller-driven market when there may be bidding wars between buyers. To account for such influential factors in the housing prices, it would be more representative to have the prices determined endogenously as an outcome of the market-clearing process.

3.2.2 Integrated Land Use, Transportation, Environment Model (ILUTE)

ILUTE is an integrated full-feedback microsimulation model with four inter-connected components: land use, location choice, auto ownership, and travel and activity (Salvini & Miller, 2005). It is a dynamic model that aims to simulate the evolution of demographics, land use, and travel within the study area over time (Miller et al., 2011). Its housing market sub-model utilizes a unique disequilibrium market-clearing approach that incorporates both game and random utility theories. It is specifically designed for a price-formation market, in which the asking prices defined by the sellers only serve as references, and the final transaction prices are a result of market interactions between buyers and sellers (Farooq & Miller, 2012; Rosenfield et al., 2013).

Prior to the market-clearing process, the asking prices of the housing units on the market in the given time step and the mobility choices of the households are determined through their respective sub-models. Detail descriptions of these sub-models can be found in Habib (2009). The asking prices are estimated for the housing units available in the market based on their dwelling attributes, macroeconomic attributes, and historical market performance. These are updated yearly as a function of actual selling prices in the previous period. The mobility model estimates whether a household decides to relocate or not to use a discrete-time random parameter model. The supply of housing units is composed of three parts: existing dwellings from households who migrated out of the region, existing dwellings from households who relocated, and new housing developments (Farooq & Miller, 2012). The methods to model new housing developments are proposed by Haider (2003), including housing starts and types and locations of the dwellings. The probabilities of choosing each alternative location are estimated using logit models, and the cumulative probabilities of all locations for each dwelling type are then calculated. A number between 0 to 1 is then randomly assigned to each new housing development to compare with the cumulative distribution value to determine its location (Farooq & Miller, 2012). This is a Monte Carlo simulation process to operationalize the housing supply model in ILUTE.

In addition, a residential location choice model based on historical housing unit sales data is used to develop a model of location utility as a function of location, dwelling, and household attributes. A unique feature of the location choice model is that it uses the status quo as a reference and incorporates the potential gains and losses of relocating when estimating the probabilities (Farooq & Miller, 2012; Habib & Miller, 2009). The random utility for a housing unit j for household i , U_{ij} , estimated through this process, takes the general form:

$$U_{ij} = \gamma R_{ij} + \beta X_{ij} + \varepsilon_{ij} = \gamma R_{ij} + \widetilde{U}_{ij} \quad (12)$$

where R_{ij} is the price i is willing to pay for j , weighted by parameter γ ($\gamma < 0$), X_{ij} is a vector of explanatory variables, with the associated vector of parameters, β , ε_{ij} is a random utility term distributed Type 1 Extreme Value, and \widetilde{U}_{ij} is the non-price component of j 's utility for household i .

Equation 12 can be inverted to yield:

$$R_{ij}^* = (U^* - \widetilde{U}_{ij})/\gamma \quad (13)$$

where R_{ij}^* is the amount that i is willing to pay for j in order to achieve reference utility U^* . It is assumed that U^* is defined as the maximum utility that household i expects it would receive if it were to buy any of the housing units in its choice set other than j at these alternative housing units' asking prices (Rosenfield, et al., 2013).

In ILUTE, the market-clearing process assumes that buyers and sellers are non-cooperative agents seeking to maximize their utilities and profits, respectively, and have limited market information (Farooq & Miller, 2012). Housing units for sale are randomly selected one at a time and are auctioned off to the highest bidder, given that this bid exceeds the seller's reservation (minimum acceptable) selling price. The actual transaction price is based on a Vickrey auction (Vickrey, 1961), in which the housing unit is sold to the highest bidder but at a price that equals the second-highest bid plus one dollar. As each housing unit is auctioned off, it is removed from the market, the purchasing household is also removed from the market, the remaining household choice sets and housing units' sets of potential bidders are updated, and the simulation proceeds to the next randomly selected housing unit to put up for bid.

A unique feature of this approach is that randomness is introduced in the generation of random bids by drawing values of the ε_{ij} terms for each housing unit j and household i . For operational purposes, the model introduces twelve sub-market cycles representing the twelve months in a year. Only a portion of the active households and housing units available in the market are cleared in a sub-market cycle.

The ILUTE model provides a market-clearing mechanism that can consider the interactions between agents and endogenously generate housing prices without enforcing an equilibrium assumption. It is versatile and can be used to simulate different market conditions. However, it is a rather complex and disaggregated model, which can be computationally intensive. In addition, the search for potential transaction prices makes this market-clearing approach highly sensitive to the asking price model (Farooq & Miller, 2012). It is important to ensure that the asking price model is well calibrated and can adequately reflect the housing market in reality. It is also worth noting that, as with any simulation model, ILUTE is path-dependent, and the sequence of clearing can affect the results.

3.2.3 SimMobility

SimMobility is a multi-scale microsimulation platform that incorporates the interaction between land-use, transportation, and activity systems (Adnan et al., 2016). It contains three simulators with different timeframes: short-term (ST), mid-term (MT), and long-term (LT). Its LT simulator is centred around housing market modules and models residential and employment locations, vehicle ownership, and land development choices (Zhu et al., 2018).

The main difference between the SimMobility housing market module and others is that it simulates a daily bidding process between active buyers and sellers. For any given day,

households that would become active in the housing market would be determined along with their affordability and choice sets of alternative housing units. A household's willingness to pay for alternative housing units is estimated through an approach built upon Lerman and Kern (1983), which is an extension of Ellickson's work. The willingness to pay is a function of household socio-economics, housing attributes, neighborhood characteristics, and accessibility measures (Basu & Ferreira, 2020). The household would only bid for the alternative housing unit that can maximize its utility surplus compared to its current dwelling. If the bid is unsuccessful and the same housing unit remains unsold, the household may adjust the bidding price and attempt to bid on the unit again on the next simulation day (Zhu et al., 2018).

The sellers place an asking price and reservation price on each housing unit for sale. If a housing unit cannot be successfully sold in a period of time, the seller may gradually reduce the asking price to the reservation price. The asking prices are estimated through a hedonic model with input variables representing housing attributes, locational characteristics (including accessibility), and temporal market dynamics. The inclusion of variables on temporal market dynamics is to capture both the macroeconomic factors and the dynamics between sellers and buyers with endogenously varying asking prices and bids. The market is cleared at the end of each simulation day, when the sellers assess all the bids for each housing unit and accept the highest under the condition that it exceeds the reservation price. The active buyers and sellers would remain in the market until a successful transaction or exit the market after a number of unsuccessful days. The sellers' and buyers' time on and off market, as well as the adjustments in asking price and bidding price, are all exogenous parameters that can be defined in the module (Zhu et al., 2018).

In addition to individual sellers, the housing supply also incorporates new housing developments. The developers' decisions are modelled as development template choices, which include the types, sizes, and densities of the new housing units. The real options theory is incorporated into the development model; therefore, the developers would only choose the template that has an expected return above a threshold and higher than all other development options (Zhu et al., 2018).

SimMobility is pioneering in attempting to simulate a daily bidding and market-clearing process. The main advantage of modeling a very temporally disaggregated (i.e., daily) housing market is the potential to capture the near-term market dynamics more realistically. It may also be able to simulate the immediate market responses to certain disruptions or policy changes such as new housing programs or interest rate variations (Zhu et al., 2018). However, the cost of such a disaggregated modeling approach is the need for an extensive amount of high-quality data and computational power. Zhu et al. (2018) also pointed out that even with applying the simulation for a "data-rich" study region, it can still be a time-consuming and delay-prone process. It can impose challenges to almost every step of the modeling process, from data collection and cleaning to model calibration and validation. In addition, it is also difficult to determine the external parameters that may have significant impacts on the market clearing process, such as time-on-market and price adjustments. Estimating these parameters through rigorous analysis and empirical studies on historical data may be preferable to simple rule-based methods.

4 Discussion of strengths, weaknesses, challenges, and future research directions

4.1 Equilibrium vs. dynamic disequilibrium assumptions

The debate between equilibrium vs. disequilibrium approaches for modeling land-use and transport systems seems to be a never-ending topic. For housing market-clearing, it is undeniable that each approach has its own advantages and drawbacks. Equilibrium-based market-clearing mechanisms are supported by microeconomic theories and allow housing prices to be endogenously generated as a result of supply-demand interaction. However, the strong assumptions about supply-demand equilibrium and agents with perfect information oversimplify the housing market, especially in situations when unlocated households or unoccupied locations exist. In addition, the instantaneous adjustment in price to match the supply and demand is not a realistic representation of the housing market. It ignores that agents in the market can impact each other's decisions; thus, time and feedback effects play an important role in the market-clearing process. These limitations of the equilibrium approach have been highlighted during the COVID-19 pandemic, during which housing markets in many urban regions experienced behavior that was very dynamic and "disequilibrated" in nature. Whether these markets will return to a "more equilibrated" state remains to be seen at the time of this paper's writing.

In contrast, dynamic disequilibrium market-clearing mechanisms account for the time and feedback effects through recursive modeling periods. This also provides the opportunity to model current choices based on past experience and possibly simulate the evolution of agents' preferences over time (Miller, 2018a). Even so, the duration of one modeling period is a nontrivial choice. Although one year is a commonly adopted time step, there is the argument that a finer time step may better represent the actual market. However, one drawback is that the recursive modeling approach introduces the risk of accumulating errors over time (Kryvobokov et al., 2013), although one experiment with ILUTE did not demonstrate this behavior (Beykaei, et al., 2014). In addition, estimating housing prices is challenging in dynamic disequilibrium models. The relaxation of the equilibrium assumption makes it difficult to endogenously generate housing prices while considering market effects and not being too computationally expensive. Hurtubia et al. (2019) proposed quasi-equilibrium market-clearing mechanisms for microsimulation models by solving a simplified approximation of the equilibrium conditions. However, the mechanism contains some simplified assumptions concerning agents' behavior, which can be improved by future research.

The timeframe of the project is also a factor to consider when choosing between equilibrium and disequilibrium approaches. For a shorter timeframe, utilizing a disequilibrium model can capture housing market dynamics. However, for projects with longer timeframes, applying the disequilibrium approach would involve simulating numerous recursive models that are path dependent and possibly prone to error cumulation. An equilibrium model can be considered for such applications if it is justifiable to assume that the housing market tends to converge to an equilibrium state over a long period of time.

4.2 Aggregation vs. disaggregation of spatial and demographic representations

A long-time criticism of equilibrium models is the aggregation of agents. They usually define homogeneous groups of households based on demographics while categorizing housing supply

through zonal attributes. Such aggregated representations may not well capture the agents' heterogeneous and nonlinear behavior, thus introducing aggregation bias into the models (Miller, 2018b). Specifically for the housing market, households have diverse preferences and tastes regarding the locations and zonal attributes related to their willingness to pay for different housing units. Although aggregation of agents is inevitable for equilibrium models, the level of aggregation is defined by the model developers and can be adjusted according to the study scopes. Some equilibrium models can be rather disaggregated relative to the others. For example, the standard aggregation in the MUSSA model includes 65 clusters of households and six types of dwellings (Martínez, 1996). However, capturing heterogeneity through cross-classification of households and locations can become computationally expensive, especially for large-scale applications.

Some researchers believe that disaggregated models through microsimulation perform better in capturing heterogeneity and providing more realistic representations of agents' behavior (Acheampong & Silva, 2015; Miller, 2018a). Most of the dynamic disequilibrium models are disaggregated microsimulation models; however, they generally require more detailed data on households' socioeconomic characteristics and attributes of the disaggregated housing units. The difficulty of collecting detailed data and the alleged "data-hungry" nature of disaggregated models have been viewed as the challenges of such models. Although the challenges persist, the rapid developments in big data science and data collection technologies such as GIS-based land-use databases provide opportunities to tackle them. In addition, the interaction between different sub-systems of a LUTI model can be better modelled at the level of individual agents. Specifically, within a microsimulation framework, the land-use sub-models can be integrated with an activity-based travel demand model.

Moreover, microsimulation models may be exploited to capture the near-term dynamics of the housing market more realistically. With disaggregated modeling of agents' interactions and enough data inputs, microsimulation models can be more powerful in simulating extreme market conditions like bidding wars or forecasting the immediate market response to sudden events such as economic shocks. SimMobility has demonstrated its ability to simulate a daily bidding and market-clearing process. However, a more important question is whether or to what extent should near-term housing market dynamics be modelled. It is without doubt that in reality, the short-term interactions and the bargaining process between buyers and sellers can influence transaction prices, and possibly the buyers' willingness to pay and relocation choices. Yet, the necessity of simulating a daily market-clearing process remains debatable. Even in real life, such a major life event as a relocation decision would require time to assess and evaluate potential alternatives, which could take weeks or even months. The temporal path dependencies and randomness persist in both real-life and microsimulation models. Without empirical studies comparing the different market clearing timeframes and assumptions, it is a nontrivial task to discuss which one can more realistically and effectively simulate the market dynamics. Meanwhile, this also reflects the key issue of how to construct a more realistic sub-market of likely buyers and sellers who would interact significantly within their expected timeframes. An ideal model should be flexible in its temporal representation so that it can be adjusted depending on the study scopes and available data inputs. The exogenous parameters linked to the disaggregation and clearing of the market need to be estimated based on historical data and calibrated for the study areas.

Regardless of equilibrium or disequilibrium approaches, disaggregation of agents is preferred for capturing their behavioral heterogeneity and more realistically representing the actual housing market-clearing process. However, as Wegener (2011) pointed out, the endless pursuit of a finer disaggregation is unnecessary. For each modeling application, an adequate level of disaggregation needs to be carefully examined and defined based on the study scope to

preserve the model's computational efficiency while providing a more realistic estimation of the housing market.

4.3 Effects of transportation accessibility on housing valuation and market clearing process

Transportation accessibility is an influential factor in housing valuation and the market-clearing process. It is often a key variable in a household's utility or willingness-to-pay function or in a hedonic price model. Despite the importance of accessibility, there is no universal approach to how it should be measured and accounted for in housing market models. For example, all of the models reviewed utilize different methods to include transportation accessibility in their valuation of housing. In the MUSSA model, accessibility measures are exogenous inputs generated from a transport model named ESTRAUS, which was developed to connect with MUSSA to form a complete LUTI model framework (Martínez, 1996). In ILUTE, simple location-based accessibility measures are included in its residential location choice model, such as distance to highways exits and subway stations (Habib & Miller, 2009). In UrbanSim, accessibility to each activity (e.g., employment) is measured as the sum of quantity for the activity at each possible destination weighted by the composite utility across all modes (i.e., log-sum of the mode choice model) for each origin-destination pair (Waddell, 2000). Such accessibility measures are often computed for selected activities and travel modes at a zonal level in the case studies (Kryvobokov et al., 2013; Waddell, 2010).

The beforementioned location-based and trip-based accessibility measures generally do not consider household heterogeneity and the impacts of trip chaining and activity scheduling on accessibility preferences. To address this limitation, activity-based accessibility (ABA) measures are developed and adopted into LUTI models like SimMobility (Ben-Akiva & Bowman, 1998; Dong et al., 2006). Such accessibility measures are generated from activity-based travel demand models and reflect the maximum utility that a household can achieve based on its activity schedules at a potential residential location (Zhu et al., 2018). Theoretically, they can be household-specific and provide a sound approach to linking an individual's short-term decisions like preferred activities and mode choices with long-term choices like residential location and vehicle ownership. However, in practice, given the limitations in data and computational power, ABA measures are often computed as a zonal average of all households. SimMobility views the integration of activity-based accessibility measures as its major advancement over other microsimulation LUTI models.

Although ABA measures seem theoretically appealing, one could also doubt whether they truly reflect a household's consideration of transportation accessibility when evaluating the price and location of alternative housing choices in reality. It is possible that the household does not think of the accessibility to all potential activity locations, but only focuses on the most significant locations like school and work. For microsimulation models, it is preferable to have household-specific representations of accessibility to major activity locations given available mode choices, whether through ABA or other measures. Such distinctions can account for households' differences in willingness to pay for an alternative housing unit. The specific approach to measuring such accessibility should be selected based on the availability of data, computational resources, and the travel demand model of the study area.

Furthermore, emerging mobility technologies have expanded the mode choices available to households. The introduction of autonomous vehicles (AV), Mobility as a Service (MaaS), and other vehicle bundle options can potentially change household preferences on transportation accessibility and thereby influence their residential location choices. A few studies examined

the ability of LUTI models to accommodate these evolving mobility tools. Hawkins and Nurul Habib (2019) critically assessed currently operational LUTI models and concluded none of them has the full capability of modeling the AV adoption process and its impacts on land-use patterns. Later, Basu and Ferreira (2020) utilized SimMobility to simulate the effects of automated mobility on housing prices, bidding results, and the choices of housing-mobility bundles. It showcased the potential of using LUTI models to explore housing market dynamics and responses to innovative technologies and relevant policies. More advanced LUTI models should consider the adoption of emerging mobility technologies and integrate them with the housing market components. Aside from the possible change in accessibility to new mobility services and facilities, they can also impact household vehicle choices in terms of both whether, when, and what type of vehicle or mobility tool to own. This leads to the question of whether household vehicle and residential relocation choices should be modelled sequentially or simultaneously given the household's budget constraints. Different modeling structures should be tested to better understand this complex decision chain and comprehend the effects of emerging mobility technologies on housing valuation and residential location choice. This remains a promising yet challenging future research direction.

4.4 Computational cost and operationalization of the existing models

Housing market clearing is a complex modeling process and requires extensive computational resources regardless of the mechanism, especially for disaggregated models. Solving for an equilibrium condition with disaggregated clusters of agents means finding a fixed-point solution for a massive system of equations, which can be challenging for both computational time and power. On the other hand, recursively simulating the market-clearing process over multiple modeling periods through dynamic disequilibrium models also involves a decent amount of computational resources and person-hours. Through an empirical comparison, Kryvobokov et al. (2013) found that the time required to complete one simulation using static equilibrium and dynamic disequilibrium models were similar. Miller (2018a) argued that directly modeling the behavior of disaggregated agents through adequate microsimulation models should be more computationally efficient. Nevertheless, the actual computational burden of an operational model is also subject to the scale of the modelled system, which is difficult to compare without an empirical study that applies both equilibrium and dynamic modeling approaches to the same system.

The continuing advancements in modern technology provide opportunities to address the challenge of high computational burdens. High-Performance Computing, such as cluster and parallelization computing can be explored to improve the computational efficiency of the models (Miller, 2018b). However, despite the advancing computer technology, some researchers still have concerns about whether the computation-intensive models can be operational and possibly utilized by various applications (Wegener, 2011).

Most operational models are only developed for a particular study area. However, given the complexity and resources required to develop such a model, it would be expected to be able to operate in different study areas for various scenario testing and analysis. This may be one of the biggest challenges the existing models will face in the future. Among the five models reviewed in this paper, UrbanSim is the most widely utilized model. It has been developed for over two decades and applied in multiple cities in the United States and Europe. Its popularity can be attributed to using an open-source platform named OPUS (i.e., Open Platform for Urban Simulation), which can generate simulations for land-use, travel demand, and traffic assignment models (Waddell et al., 2006). OPUS makes UrbanSim an easily available and

flexible model (Renner et al., 2015). In addition, compared to other disequilibrium models like ILUTE and SimMobility, its housing market-clearing mechanism is relatively simple to implement.

In contrast, the ILUTE model has only been applied to the Greater Toronto and Hamilton Area in Ontario, Canada. Its detailed microsimulation structure and relatively complex market-clearing process may be the main challenge for calibrating and applying it to other study areas. The same situation also applies to the SimMobility model. So far, the studies related to SimMobility found in the literature are all tested in Singapore. Its daily bidding and market-clearing process required even more data and effort to calibrate for other study areas. As for equilibrium models, both MUSSA and RELU-TRAN have been tested in multiple study areas (Anas, 2013; Renner et al., 2015).

Waddell (2011) provided an in-depth discussion on the challenges encountered while operationalizing UrbanSim for multiple projects. This paper highlights the challenges of moving academic models into operational agency settings in terms of transparency, validity, computational performance of the models, etc. There seems to be a certain trade-off between the complexity, validity, and operationalization of a model. Specifically for market-clearing mechanisms, how to balance the complexity of the model and the behavioral validity of the simulated market effects remains a challenge to be addressed in the future.

4.5 Future research directions

Despite the continuing development of LUTI models, very few studies have attempted to compare equilibrium and disequilibrium models by applying them to the same study area. One example is Kryvobokov et al. (2013), who compared a static equilibrium model named Pirandello with UrbanSim. They evaluated the long-term effects of a tolling project and concluded that the two models could generate comparable empirical results. However, this is an isolated case and cannot be viewed as a definite conclusion. As Simmonds et al. (2013) point out, there are competing hypotheses about how well equilibrium models can estimate the effects of external shocks and policy changes. The existing comparative studies mostly focus on particular models or small-scale empirical applications. To better evaluate the performance of the two approaches, a more thorough study is needed to systematically compare them and understand their applicability under different study periods, initial conditions, model assumptions, and external shocks. Collaborations and communications between researchers from different institutes and regions are encouraged to provide a more comprehensive comparison of the models from different perspectives. Such a systemic comparison is important for housing market-clearing mechanisms, since they are generally the core of the land-use models. Specifically, the comparison should analyze how well the models capture and represent the agents' behavior in a market with a supply or demand surplus. Moreover, the comparison should also explore the models' abilities to simulate the effects of policies on adjusting housing prices and balancing supply and demand. For example, financial policies like change in interest rates can directly influence the housing prices. The models should be able to reflect the short- and long-term effects of such policies for professionals to further study their implications.

In addition, it is recommended to study further the behavior of sellers and buyers in the housing market, particularly in extreme disequilibrium market conditions such as seller-driven and buyer-driven markets. The traditional approach treats the housing market clearing as a matching process. However, such an approach cannot fully address the impacts of the buyers' and sellers' behavior on the market and housing prices. Specifically, the sellers' behavior is

relatively less investigated than the buyers' behavior. More sophisticated studies are needed to answer the following questions and their association with the market-clearing process: 1) how the sellers' preferred prices are influenced by the market, 2) how long would the sellers stay in the market to achieve the preferred prices, and 3) whether they would leave and re-enter the market if the preferred prices cannot be reached.

A potential challenge for the abovementioned research directions is the lack of detailed data on sellers' and buyers' behavior. Generally, the data available for housing market models are land-use data and general households' socioeconomic information. However, specific data that may be useful to analyze agents' behavior are rarely available, such as buyers' search process, mortgage options, sellers' purpose of selling and maximum expected time-on-market, etc. One way to address this challenge is to conduct targeted surveys on recent homebuyers and sellers to gather information and understand factors that can influence their locational preferences and behavior in the market. The potential to utilize real estate transaction data in housing market models can also be explored.

5 Conclusions

This paper offers a comprehensive review of the housing market-clearing mechanisms utilized by current LUTI models. Two equilibrium models – MUSSA and RELU-TRAN and three dynamic disequilibrium models – UrbanSim, ILUTE, and SimMobility are reviewed and discussed in their strengths and weaknesses. The equilibrium approach is relatively simple and elegant for clearing the market based on microeconomic theory. Housing prices are estimated as a result of the market-clearing process. However, its assumptions of demand-supply equilibrium, instantaneous price adjustment, and agents with perfect information oversimplify the housing market.

In contrast, dynamic disequilibrium models incorporate time and feedback effects and do not enforce the equilibrium condition. Agents are often assumed to have limited information about the market. Dynamic disequilibrium models are more realistic and better represent the disaggregated agents. However, depending on the market-clearing mechanisms, some may fail to account for the market effects when estimating housing prices. In contrast, others may suffer from complexity and have high computational costs.

Through a discussion of the aggregation level of spatial and demographic representations, the review concludes that the disaggregation of agents can better capture their heterogeneity and nonlinear behavior. An adequate level of disaggregation needs to be examined based on the scope of the application. Although disaggregation may be challenging due to data constraints and high computational costs, the continuing developments in advanced data collection and computing technologies provide opportunities to address such issues. The effects of transportation accessibility and emerging mobility technologies on modeling housing prices and the market-clearing process are also discussed. It is concluded that specific approaches to measuring accessibility should be selected based on the availability of data, computational resources, and the travel demand model of the study area. The review suggests that future research should pay more attention to the impacts of sellers' and buyers' behavior on housing prices and the marketing-clearing process. A more sophisticated study on the agents' behavior in seller-driven and buyer-driven markets is needed. In addition, to better evaluate the performance of the equilibrium and disequilibrium approaches, a systematic comparison of different market-clearing mechanisms is needed to understand their applicability under different study periods, initial conditions, model assumptions, and external shocks.

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References

- Acheampong, R. A., & Silva, E. A. (2015). Land use–transport interaction modeling: A review of the literature and future research directions. *Journal of Transport and Land Use*, 8(3), 11–38.
- Adnan, M., Pereira, F., Lima Azevedo, C., Basak, K., Lovric, M., Raveau, S., ... & Ben-Akiva, M. (2016, January 11). *SimMobility: A multi-scale integrated agent-based simulation platform*. Paper presented at the Transportation Research Board 95th Annual Meeting, January 10–14, Washington, DC.
- Alonso, W. (1960). A theory of the urban land market. *Papers in Regional Science*, 6(1), 149–157. <https://doi.org/10.1111/j.1435-5597.1960.tb01710.x>
- Anas, A. (2013). A summary of the applications to date of RELU-TRAN, a microeconomic urban computable general equilibrium model. *Environment and Planning B: Planning and Design*, 40(6), 959–970. <https://doi.org/10.1068/b38206>
- Anas, A., & Arnott, R. J. (1991). Dynamic housing market equilibrium with taste heterogeneity, idiosyncratic perfect foresight, and stock conversions. *Journal of Housing Economics*, 1(1), 2–32. [https://doi.org/10.1016/S1051-1377\(05\)80023-3](https://doi.org/10.1016/S1051-1377(05)80023-3)
- Anas, A., & Arnott, R. J. (1993). Development and testing of the Chicago prototype housing market model. *Journal of Housing Research*, 4(1), 73–129.
- Anas, A., & Liu, Y. (2007). A regional economy, land use, and transportation model (relu-Tran©): Formulation, algorithm design, and testing*. *Journal of Regional Science*, 47(3), 415–455. <https://doi.org/10.1111/j.1467-9787.2007.00515.x>
- Basu, R., & Ferreira, J. (2020). A LUTI microsimulation framework to evaluate long-term impacts of automated mobility on the choice of housing-mobility bundles. *Environment and Planning B: Urban Analytics and City Science*, 47(8), 1397–1417. <https://doi.org/10.1177/2399808320925278>
- Ben-Akiva, M., & Bowman, J. L. (1998). Integration of an activity-based model system and a residential location model. *Urban Studies*, 35(7), 1131–1153. <https://doi.org/10.1080/0042098984529>
- Beykaei, S. A., Miller, E. J., & Vaughan, J. (2014). *Testing the stochastic behavior of a microsimulation model of demographic and housing market dynamic*. Paper presented at the Second World Symposium in Transport and Land Use Research, Delft, June 26, 2014.
- Cordera, R., Ibeas, Á., dell’Olio, L., & Alonso, B. (Eds.). (2017). *Land use–transport interaction models*. Boca Raton, FL: CRC Press. <https://doi.org/10.1201/9780203711811>
- de la Barra, T., Pérez, B., & Vera, N. (1984). TRANUS-J: Putting large models into small computers. *Environment and Planning B: Planning and Design*, 11(1), 87–101. <https://doi.org/10.1068/b110087>
- Dong, X., Ben-Akiva, M. E., Bowman, J. L., & Walker, J. L. (2006). Moving from trip-based to activity-based measures of accessibility. *Transportation Research Part A: Policy and Practice*, 40(2), 163–180. <https://doi.org/10.1016/j.tra.2005.05.002>
- Ellickson, B. (1981). An alternative test of the hedonic theory of housing markets. *Journal of Urban Economics*, 9(1), 56–79. [https://doi.org/10.1016/0094-1190\(81\)90048-6](https://doi.org/10.1016/0094-1190(81)90048-6)
- Farooq, B., & Miller, E. J. (2012). Towards integrated land use and transportation: A dynamic disequilibrium based microsimulation framework for built space markets. *Transportation Research Part A: Policy and Practice*, 46(7), 1030–1053. <https://doi.org/10.1016/j.tra.2012.04.005>
- Fujita, M. (1989). *Urban economic theory: Land use and city size*. Cambridge, UK: Cambridge University Press.
- Guevara, C. A., & Ben-Akiva, M. (2006). Endogeneity in residential location choice models. *Transportation Research Record*, 1977(1), 60–66. <https://doi.org/10.1177/0361198106197700108>
- Habib, M. A. (2009). *Microsimulating residential mobility and location choice processes within an integrated land use and transportation modelling system* (PhD thesis), University of Toronto, Toronto, Ontario, Canada.
- Habib, M. A., & Miller, E. J. (2009). Reference-dependent residential location choice model within a relocation context. *Transportation Research Record*, 2133(1), 92–99. <https://doi.org/10.3141/2133-10>

- Haider, M. (2003). *Spatio-temporal modelling of housing starts in the Greater Toronto Area*. Toronto, Ontario: University of Toronto.
- Hawkins, J., & Nurul Habib, K. (2019). Integrated models of land use and transportation for the autonomous vehicle revolution. *Transport Reviews*, 39(1), 66–83. <https://doi.org/10.1080/01441647.2018.1449033>
- Hunt, J. D., & Abraham, J. E. (2005). Design and implementation of PECAS: A generalized system for allocating economic production, exchange and consumption quantities. In M. E.H. Lee-Gosselin & S. T. Doherty (Eds.), *Integrated land-use and transportation models* (pp. 253–273). Bingley, UK: Emerald Group Publishing Limited. <https://doi.org/10.1108/9781786359520-011>
- Hunt, J. D., Kriger, D. S., & Miller, E. J. (2005). Current operational urban land-use–transport modelling frameworks: A review. *Transport Reviews*, 25(3), 329–376. <https://doi.org/10.1080/0144164052000336470>
- Hurtubia, R. (2012). *Discrete choice and microsimulation methods for agent-based land use modeling*.
- Hurtubia, R., Martínez, F., Flötteröd, G., & Bierlaire, M. (2010). *Comparative analysis of hedonic rents and maximum bids in a land-use simulation context*. 20.
- Hurtubia, R., Martinez, F. J., & Bierlaire, M. (2019). A quasi-equilibrium approach for market clearing in land use microsimulations. *Environment and Planning B: Urban Analytics and City Science*, 46(3), 445–468. <https://doi.org/10.1177/2399808317719071>
- Kryvobokov, M., Chesneau, J.-B., Bonnafous, A., Delons, J., & Piron, V. (2013). Comparison of static and dynamic land use–transport interaction models: Pirandello and UrbanSim applications. *Transportation Research Record*, 2344(1), 49–58. <https://doi.org/10.3141/2344-06>
- Lerman, S. R. (1976). Location, housing, automobile ownership, and mode to work: A joint choice model. *Transportation Research Record*, 610, 6–11. <https://trid.trb.org/view/53665>
- Lerman, S. R., & Kern, C. R. (1983). Hedonic theory, bid rents, and willingness-to-pay: Some extensions of Ellickson’s results. *Journal of Urban Economics*, 13(3), 358–363. [https://doi.org/10.1016/0094-1190\(83\)90039-6](https://doi.org/10.1016/0094-1190(83)90039-6)
- Lopes, A. S., Loureiro, C. F. G., & van Wee, B. (2019). LUTI operational models review based on the proposition of an *a priori* ALUTI conceptual model. *Transport Reviews*, 39(2), 204–225. <https://doi.org/10.1080/01441647.2018.1442890>
- Martínez. (1992). The bid—choice land-use model: An integrated economic framework. *Environment and Planning A: Economy and Space*, 24(6), 871–885. <https://doi.org/10.1068/a240871>
- Martínez, F. (1996). MUSSA: Land use model for Santiago City. *Transportation Research Record*, 1552(1), 126–134. <https://doi.org/10.1177/0361198196155200118>
- Martínez, F. (2018). *Microeconomic modeling in urban science*. Cambridge, MA : Academic Press.
- Martínez, F., Aguila, F., & Hurtubia, R. (2009). The constrained multinomial logit: A semi-compensatory choice model. *Transportation Research Part B: Methodological*, 43(3), 365–377. <https://doi.org/10.1016/j.trb.2008.06.006>
- Martínez, F., & Donoso, P. (2010). The MUSSA II land use auction equilibrium model. In F. Pagliara, J. Preston, & D. Simmonds (Eds.), *Residential location choice: Models and applications* (pp. 99–113). Berlin: Springer. https://doi.org/10.1007/978-3-642-12788-5_5
- Martínez, F., & Hurtubia, R. (2006). Dynamic model for the simulation of equilibrium status in the land use market. *Networks and Spatial Economics*, 6(1), 55–73. <https://doi.org/10.1007/s11067-006-7685-4>
- McFadden, D. (1977). Modelling the choice of residential location. *Transportation Research Record*, 673, 72–77.
- Miller, E. J. (2018a). The case for microsimulation frameworks for integrated urban models. *Journal of Transport and Land Use*, 11(1), 1025–1037. <https://doi.org/10.5198/jtlu.2018.1257>
- Miller, E. J. (2018b). Viewpoint: Integrated urban modeling: Past, present, and future. *Journal of Transport and Land Use*, 11(1), 387–399. <https://doi.org/10.5198/jtlu.2018.1273>
- Miller, E. J., Farooq, B., Chingcuanco, F., & Wang, D. (2011). Historical validation of integrated transport–land use model system. *Transportation Research Record*, 2255(1), 91–99. <https://doi.org/10.3141/2255-10>

- Moeckel, R. (2016). Constraints in household relocation: Modeling land-use/transport interactions that respect time and monetary budgets. *Journal of Transport and Land Use*, 10(1), 211–228. <https://doi.org/10.5198/jtlu.2015.810>
- Moeckel, R., Llorca Garcia, C., Moreno Chou, A. T., & Okrah, M. B. (2018). Trends in integrated land use/transport modeling: An evaluation of the state of the art. *Journal of Transport and Land Use*, 11(1), 463–476. <https://doi.org/10.5198/jtlu.2018.1205>
- Moeckel, R., Schwarze, B., Spiekermann, K., & Wegener, M. (2007). Simulating interactions between land use, transport and environment. In *Proceedings of the 11th World Conference on Transport Research*. Berkeley, CA: University of California, Berkeley.
- Renner, C. Z., Nicolai, T. W., & Nagel, K. (2015). Agent-based land use transport interaction modeling: State of the art. In *Integrated transport and land use modeling for sustainable cities* (p. 31). Vaud, Switzerland: EPFL Press.
- Rose, J. M., & Martínez, F. J. (2007). Towards a land-use and transport interaction framework. In D. A. Hensher & K. J. Button (Eds.), *Handbook of transport modelling* (Vol. 1, pp. 181–201). Bingley, UK: Emerald Group Publishing Limited. <https://doi.org/10.1108/9780857245670-009>
- Rosen, S. (1974). Hedonic prices and implicit markets: Product differentiation in pure competition. *Journal of Political Economy*, 82(1), 34–55. <https://doi.org/10.1086/260169>
- Rosenfield, A., Chingcuanco, F., & Miller, E. J. (2013). Agent-based housing market microsimulation for integrated land use, transportation, environment model system. *Procedia Computer Science*, 19, 841–846. <https://doi.org/10.1016/j.procs.2013.06.112>
- Salvini, P., & Miller, E. J. (2005). ILUTE: An operational prototype of a comprehensive microsimulation model of urban systems. *Networks and Spatial Economics*, 5(2), 217–234. <https://doi.org/10.1007/s11067-005-2630-5>
- Simmonds, D., Waddell, P., & Wegener, M. (2013). Equilibrium versus dynamics in urban modelling. *Environment and Planning B: Planning and Design*, 40(6), 1051–1070. <https://doi.org/10.1068/b38208>
- Vickrey, W. (1961). Counterspeculation, auctions, and competitive sealed tenders. *The Journal of Finance*, 16(1), 8–37. <https://doi.org/10.1111/j.1540-6261.1961.tb02789.x>
- Waddell, P. (2000). A behavioral simulation model for metropolitan policy analysis and planning: Residential location and housing market components of urbansim. *Environment and Planning B: Planning and Design*, 27(2), 247–263. <https://doi.org/10.1068/b2627>
- Waddell, P. (2002). UrbanSim: Modeling urban development for land use, transportation, and environmental planning. *Journal of the American Planning Association*, 68(3), 297–314. <https://doi.org/10.1080/01944360208976274>
- Waddell, P. (2010). Modeling residential location in UrbanSim. In F. Pagliara, J. Preston, & D. Simmonds (Eds.), *Residential location choice* (pp. 165–180). Berlin: Springer. https://doi.org/10.1007/978-3-642-12788-5_8
- Waddell, P. (2011). Integrated land use and transportation planning and modelling: Addressing challenges in research and practice. *Transport Reviews*, 31(2), 209–229. <https://doi.org/10.1080/01441647.2010.525671>
- Waddell, P., Borning, A., Ševčíková, H., & Socha, D. (2006). Opus (Open Platform for Urban Simulation) and UrbanSim 4. *Proceedings of the 2006 National Conference on Digital Government Research*, 360–361. <https://doi.org/10.1145/1146598.1146702>
- Wang, Y., & Wu, L. (2010). Integrated land-use and transportation models. *Paper presented at the 18th International Conference on Geoinformatics*, Peking University, Beijing, China, June 19–20. <https://doi.org/10.1109/GEOINFORMATICS.2010.5567835>
- Wegener, M. (2011). From macro to micro—how much micro is too much? *Transport Reviews*, 31(2), 161–177. <https://doi.org/10.1080/01441647.2010.532883>
- Wegener, M., & Fuerst, F. (2004). *Land-use transport interaction: State of the art* (SSRN Scholarly Paper ID 1434678). *Social Science Research Network*. Retrieved from <https://doi.org/10.2139/ssrn.1434678>
- Zhu, Y., Diao, M., Ferreira, J., & Zegras, P. C. (2018). An integrated microsimulation approach to land-use and mobility modeling. *Journal of Transport and Land Use*, 11(1), 633–659.

Ziemke, D., Nagel, K., & Moeckel, R. (2016). Towards an agent-based, integrated land-use transport modeling system. *Procedia Computer Science*, 83, 958–963.
<https://doi.org/10.1016/j.procs.2016.04.192>